Mathematical Morphology for Design and Manufacturing

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Introduction

- In general geometric models, the design of objects is usually separated from the manufacture
- At the present, two general solid modeling representations dominate: B-REP (Boundary representation) and CSG (Constructive Solid Geometry)
- They have some drawbacks:
  - The design of complex objects is usually complicated.
    - CSG: limited number of primitives for free surfaces design
    - B-REP: modification of free surfaces hardly intutitive
  - A secondary superficial model is needed (triangulation).
    - CSG uses volumes instead of surfaces
    - B-REP uses heterogeneous representations for different surfaces
  - They almost completely separate the design process from the manufacturing process.
    - More freedom for the designer
    - Subsequent complex operation are required to obtain the machining paths
    - The final object may not accurately represent the original design

Introduction

- The Surface-Trajectory model describes the objects through manufacturing operations, but a secondary surface representation is also required

- We propose a model based on Mathematical Morphology:
  - An arithmetic support to the generation of machining paths is provided
  - Since every object consists in point sets, no operation between solids or surfaces is needed
  - It integrates the machining path calculation and 2D/3D visualization of the manufactured objects

Morphological Model of Representation

- Mathematical Morphology is based on Set Theory:
    - Sets represent object shapes in an *n*-dimensional space
    - Morphological operations represent geometric relations between points
    - Analogy between design and machining: the specification of pieces and tools is achieved through the use of point sets and operations describing processes of cutting and reconstruction (erosion and dilation)
- The classical morphological model does not include any order restriction
- For manufacturing purposes, the order of the morphology operations is important since it will represent the tool trajectory
- Therefore, an order relation between set elements will be incorporated so that an operations sequence could be established.
- Moreover, only the boundary of operations will be considered, since the machining tool always attacks the material from the outside.

Morphological Model of Representation

- Let E be the domain where the sets are defined ($E \equiv Rn$)
- Let $X \subseteq E$ be a subset of E (a 2D or 3D object)
- $Fr(X)$ is a function defined to relate a set to its border, so that all the point in the object contour are obtained

$$Fr: \mathrm{P}(E) \rightarrow \mathrm{P}(E)$$

$$Fr(X) = \{p \in X, p \text{ is a contour point of } X\}$$

- The erosion operation is defined as the place of center positions (y) of the structuring element B when it is forced to be inside X:

$$X \odot B = \{y \in E, B_y \subseteq X\}$$

- For solid objects (without inner holes), the contour of the erosion is the center positions of the structuring element when it touches the inner frontier of the object

$$Fr(X \odot B) = \{y \in E: B_y \subseteq X \ \wedge \ B_y \cap Fr(X) \neq \varnothing\}$$

Instant basic operations

- A morphological operation will be divided into a sequence of unitary or *basic* operations.
- The sequence will guarantee the resulting order of the whole operation.
- Every basic operation corresponds to a particular tool position along a trajectory, so we call them *instant basic operations.*
- *The instant basic operator* $\odot_{\Gamma(k)}$ for instant k is defined:

Homogeneous transform matrix in R^{n+1} x R^{n+1} for a specific k real value

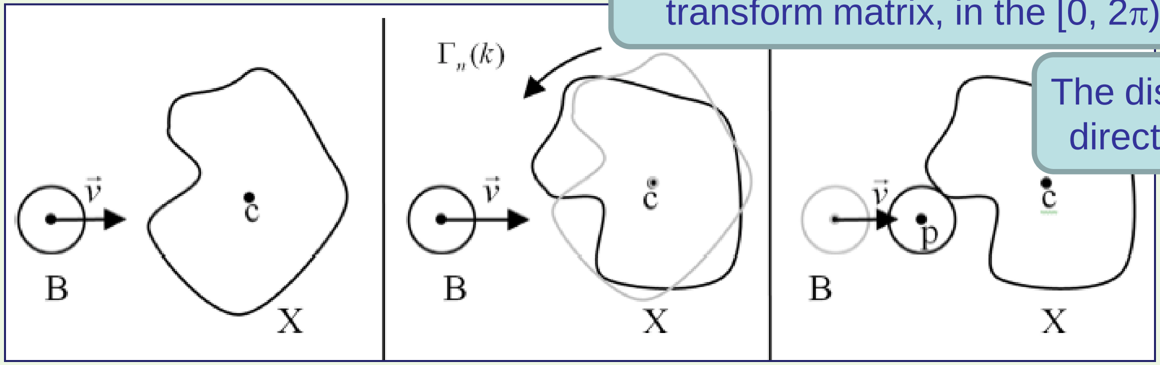
Euclidean distance between str. elem. and the transformation of X computed in the direction of $\mathbf{v}$ vector

$$X \odot_{\Gamma(k)} B = y \in E : y = dist_{\mathbf{v}}(B, X \bullet \Gamma(k)) \cdot \mathbf{v} \;\wedge\; B_y \cap X \neq \emptyset$$

- It obtains the str.elem. center position when it touches the X boundary following the $\mathbf{v}$ direction.

k could be the number of degrees in the transform matrix, in the $[0, 2\pi)$ range

The distance in the $\mathbf{v}$ direction is applied



Instant basic operations

- For different ordered *k* real values (< relation), a new set of str.elem. center positions is obtained touching the *X* boundary.
- The only required geometric calculations are a homogeneous transform and the Euclidean distance.
- From the instant erosion operation $\odot_{\Gamma(k)}$, a new operation is defined so that it fulfills the condition of including the str.elem. in the original set.
- *Erosion with trajectory* is defined as the points set obtained after repeated application of $\odot_{\Gamma(k)}$ for [0, 1]

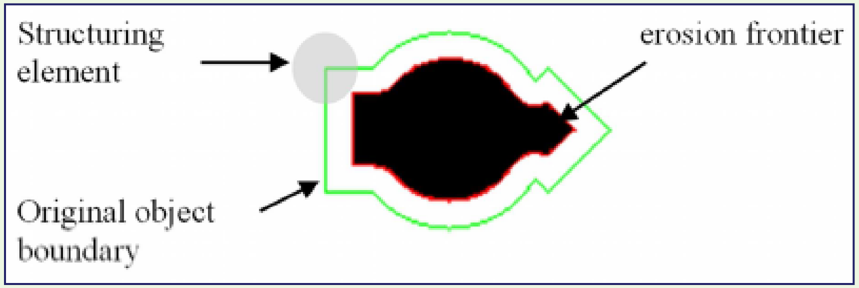
$$X \odot_{\Gamma} B = \bigcup_{k \in [0,1]} (X \odot_{\Gamma(k)} B) = \{\, y \in E \colon B_y \cap Fr(X) \neq \varnothing \wedge B_y \subseteq X \,\}$$

- If *k* takes succesive values in the interval, the boundary of the complete erosion if obtained.
- The boundary of the morphological erosion and the erosion with trajectory will coincide:

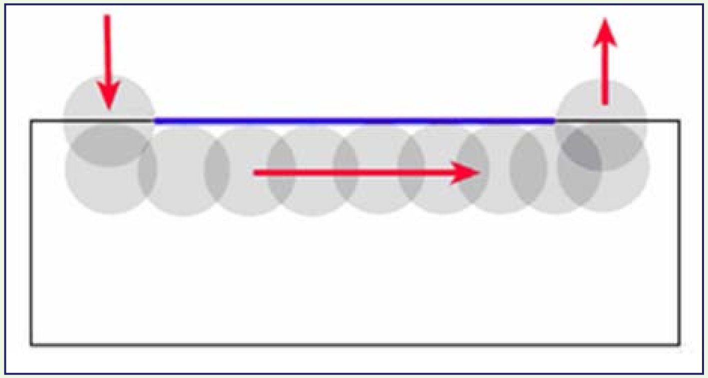
$$X \odot_{\Gamma} B = Fr(X \odot B) \subseteq X \odot B$$

Instant basic operations

- An ordered succesion of *k* values in [0,1] will lead to obtain an ordered collection of str.elem. centers as the movement of transform Γ is performed.



- The machining process (milling operation by using a tool) can be interpreted as a morphological operation of erosion, where a str.elem. touches an object following a given direction
- However, machining processes are not limited to closed trajectories since they also include open or partial paths (entry and exit of the tool)



Instant basic operations

- Therefore, a new operation of partial erosion will be defined as a subset of the complete erosion:

$$X \odot_{P\Gamma} B \subseteq X \odot_{\Gamma} B$$

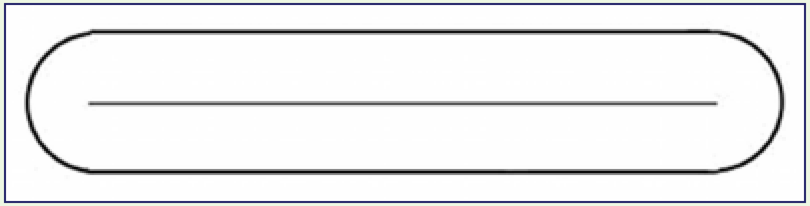
- However, it has not a direct equivalence in the machining process.

- It will be redefined as a partial regularized operation, adapted to the implicit requirements of the machining process.

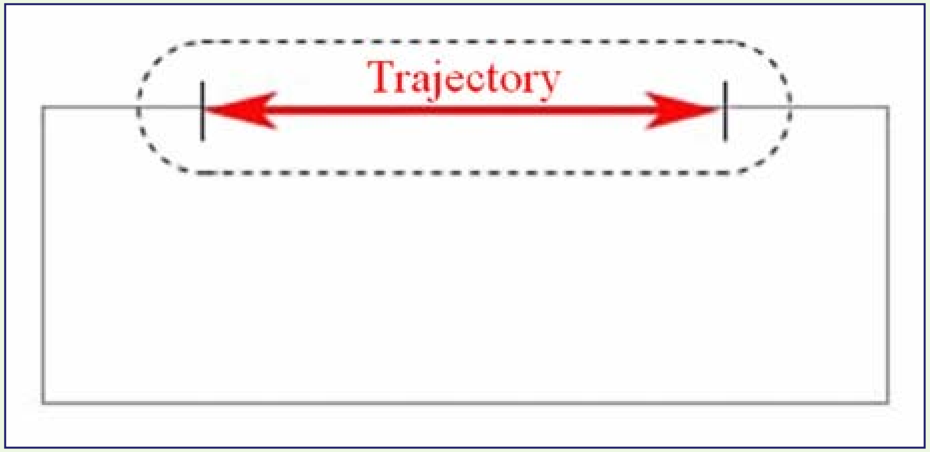
Instant basic operations

- The dilation is defined as the place of the center positions of the str.elem. when it touches *X*.

$$X \oplus B = \{y \in E, B_y \cap X \neq \emptyset\}$$



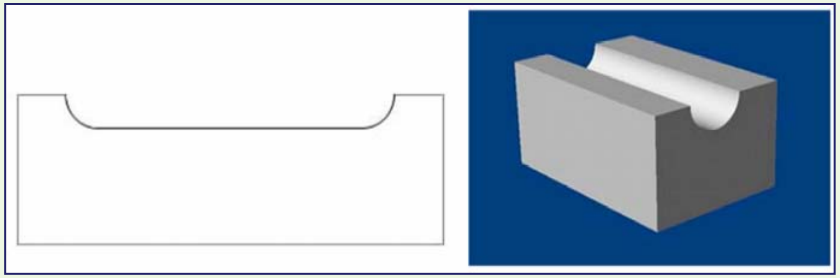
- From the machining point of view, the regularized partial erosion can be defined as the subtraction of the dilated trajectory from the object to erode.



Instant basic operations

- Given *X*, *B*, and an erosion trajectory *T*, the regularized partial erosion of X is the result of subtracting the dilation of T with the radius of B from X.

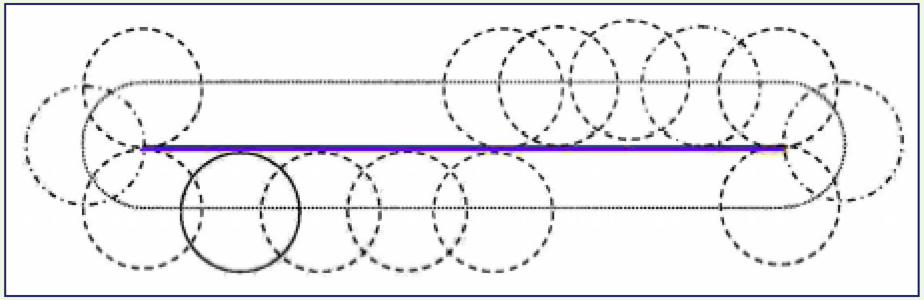
$$X \Theta_{\Gamma} B = X\sim(T \odot\oplus_{\Gamma} B) \; ; \; T \subseteq Fr(X)$$



Secondary 2D model of representation

- The only information the model requires is based on erosion paths and object.
- For representing the result (frontier) of regularized partial erosions on screen, it is necessary to provide a *discrete* model of representation.

- The dilation of the erosion trajectory of a tool on a segment of a 2D object can be geometrically described from the calculation of the parallel on both sides of the lines, using the tool radius and the parallel distance.



- The outer part of the dilation has no influence on the erosion since it does not contact the eroded element.

Secondary 2D model of representation

- The result of this subtraction is given by the subsetof points belonging to the object that does not contact the str.elem. as well as the points derived from the dilation of the trajectory.
- Since it is an infinite set of points, a discretization is needed for represent it.
- This meshing process will be naturally obtained thanks to the order relation previously introduced.
- For representing a 2D dilation, the solution just consists in discretizing the *X* set in n_{seg} segments, and calculating every point *P* in the parallel of each segment $p_i q_i$ (number of segment *i* defined by its initial and final points)

R is the tool radius

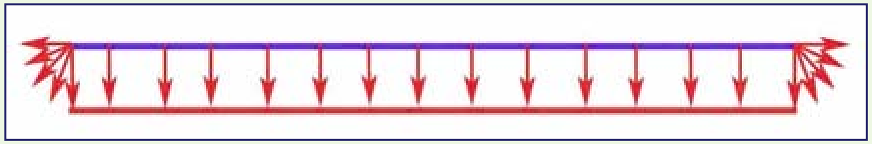
$$P = \left\{ \begin{array}{l} p'_{i_j} = p_{i_j} + R \cdot \dfrac{(p_i y - q_i y, q_i x - p_i x)}{\sqrt{(q_i x - p_i x)^2 + (q_i y - p_i y)^2}} \\ : p_{i_j} \in X; \forall j = 0, \ldots, d_i; \forall i = 0, \ldots, n_{seg} \end{array} \right\}$$

p_{ij} is every discrete point for segment $p_i q_i$

d_i is the number of discrete points on segment piqi

Secondary 2D model of representation

- At the end points, an angular sweep of normals is done so as to obtain the dilation of the trajectory.



- The sweep (*Bj*) for an extreme *pj* is defined as the point set resulting from rotating *n* times (precision) the point *p*' through an angle α_i

$$Bj = \{p1, p2, \dots, pn\} = U_{i=1,\,2..,n}(Rot(p'j,\ \alpha_i))$$

$$\alpha_i = (i \cdot \alpha)/n;\ i = 1,\dots,\ n;\ \alpha\ = ang(vector\ n,\ vector\ p)$$

- Taking into account the point set of the parallel (P) and the point set obtained through a *t* steps sweeping (CB), the dilation is defined:

$$Ep2D = P\ U\ B;\ CB = U_{j=\,0,..,t}(Bj)$$

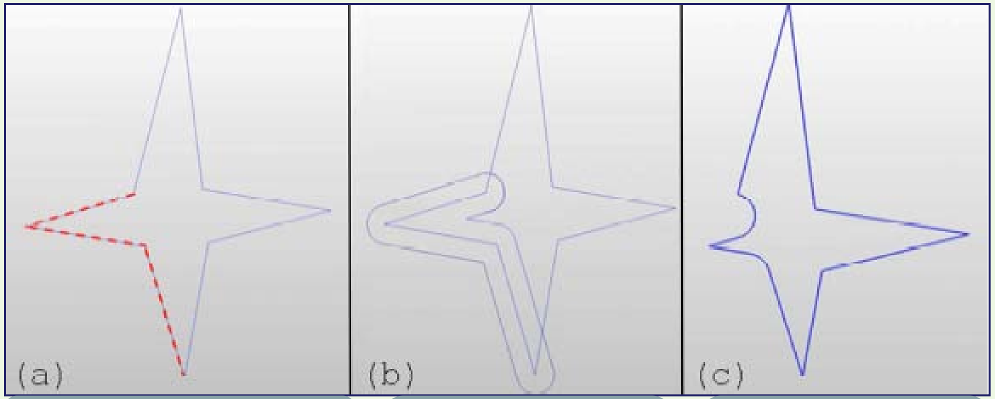
Secondary 2D model of representation

- Algorithm for dilation in 2D:

```
segments.Discretize(Points)
for each point ∊ Points do
      if points.IsTrajectoryBeginning( ) or point.IsTrajectoryEnd( )do
            Points = point.NormalSwept( )
            2DErosionP.AddPoints( Points )
      else
            if point.NeighbourAngle( )>0 do
                  Points = point.NormalSwept( )
                  2DErosionP.AddPoints( Points )
            else
                  point = point.Parallel( )
                  2DErosionP.AddPoint( point )
            endif
      endif
endfor
```

Secondary 2D model of representation

- The partial erosion is the subtraction of the obtained dilation from the object to be erode, and the result is the frontier of the machined object.

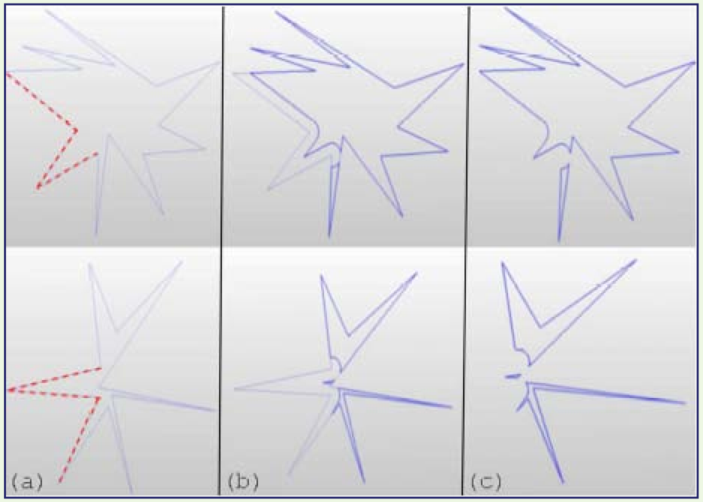


Path to be eroded

Dilation of the trajectory

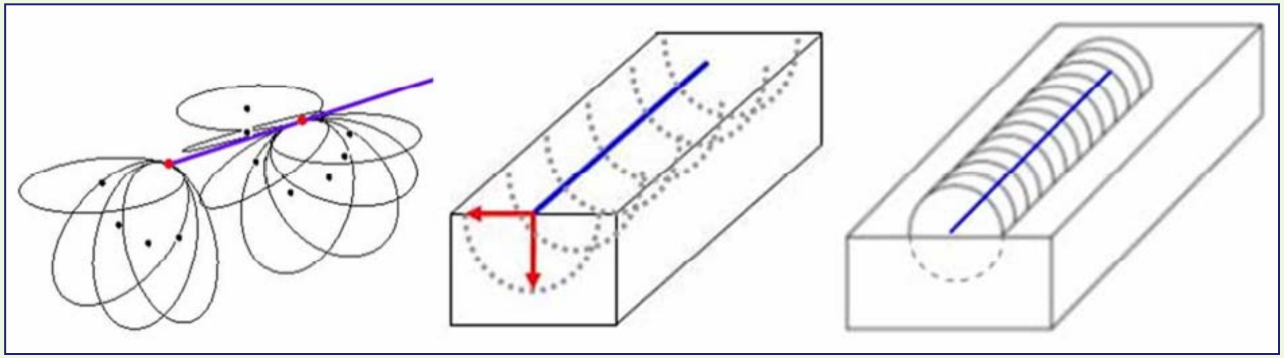
Regularized partial erosion

- Some cases of discontinuities:



Secondary 3D model of representation

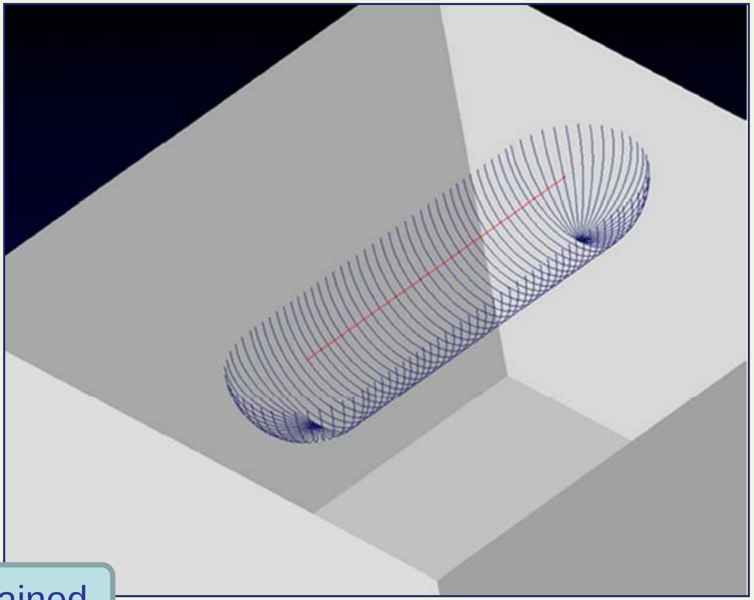
- In 3D, two senses (side and front) of rotation must be calculated, giving the dilation a 3D treatment through a cylindrical shape.
- For representing a 3D dilation, a lateral rotational sweep is done for each point belonging to the segment.
- For extreme points, a frontal rotation sweep is also done.
- The lateral sweep gives perpendicular arches and the frontal one produces hemispheres.
- As in 2D, only the point inside the element to be eroded participate in the erosion.



Secondary 3D model of representation

- Algorithm for partial erosion in 3D

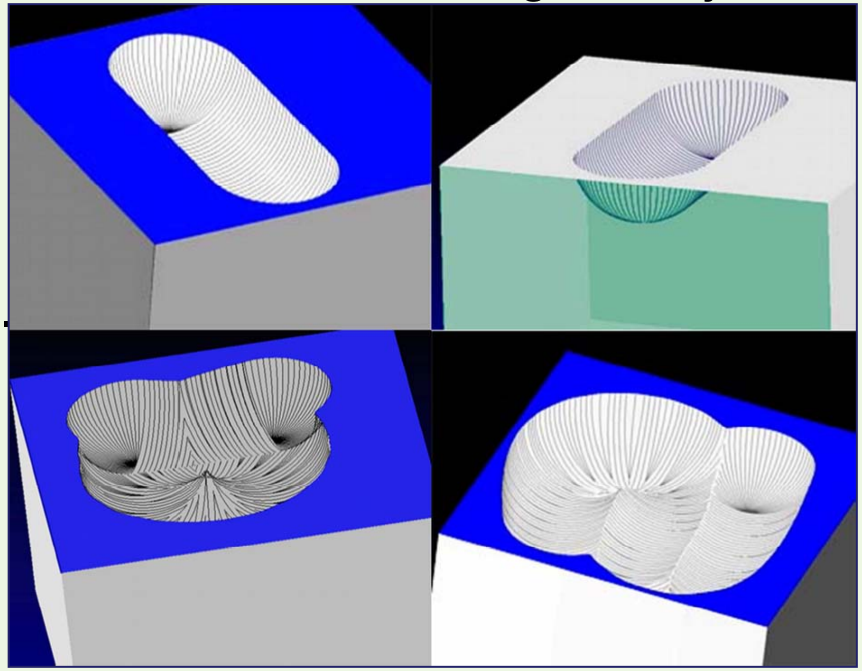
```
Trajectory.Discretize(Points)
for each point ∊ Points do
      if point.IsTrajectoryBeginning( ) or point.IsTrajectoryEnd( )do
                 Points = point.SideSwept( )
                 3DErosionP.AddPoints( points )
                 points = point.FrontSwept( )
                 3DErosionP. AddPoints( points )
      else
                 points = point.SideSwept( )
                 3DErosionP. AddPoints( points )
      endif
endfor
```



The dilation frontier is obtained

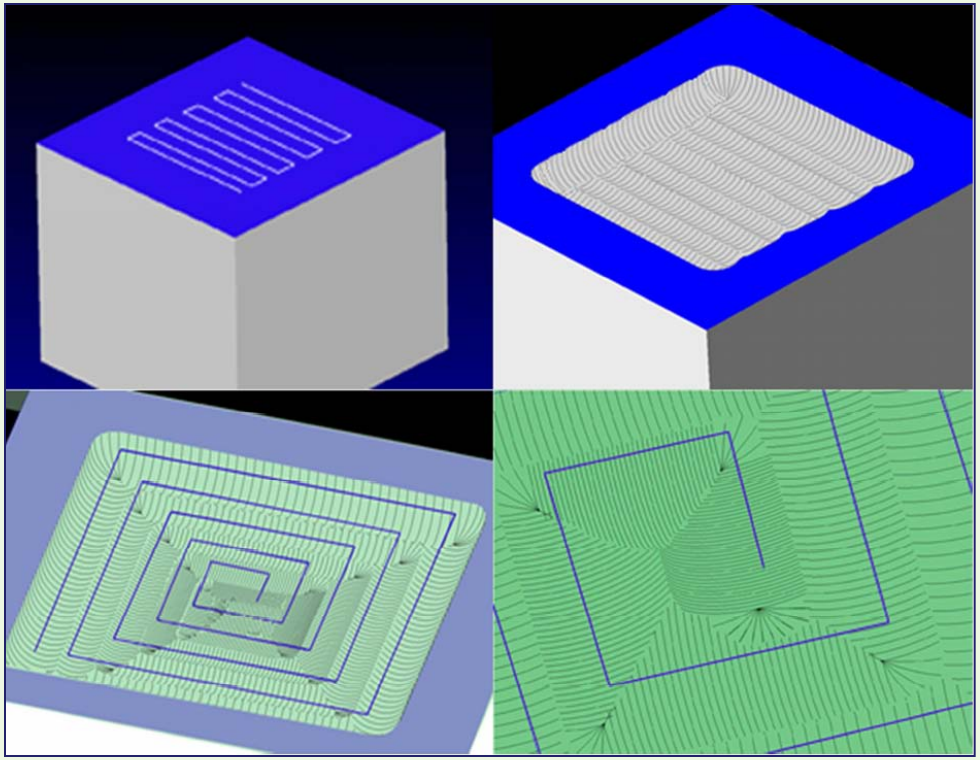
Secondary 3D model of representation

- The whole regularized erosion needs subtracting the dilation frontier from the frontier of the original object.
- The sum of all the surfaces will constitute a single solid-like object :
    - The surfaces ob the initial shape not involved in the erosion
    - Some portions of the surfaces of the initial shape involved in erosion (cut)
    - The surface of the dilation generated from the arches
- The first step consists in obtaining the cut surface from the original object and the erosion trajectory.
- For this step, the erosion contour is used as the profile to cut the original surface, so that the obtained cut surface will be a portion of the final eroded object.



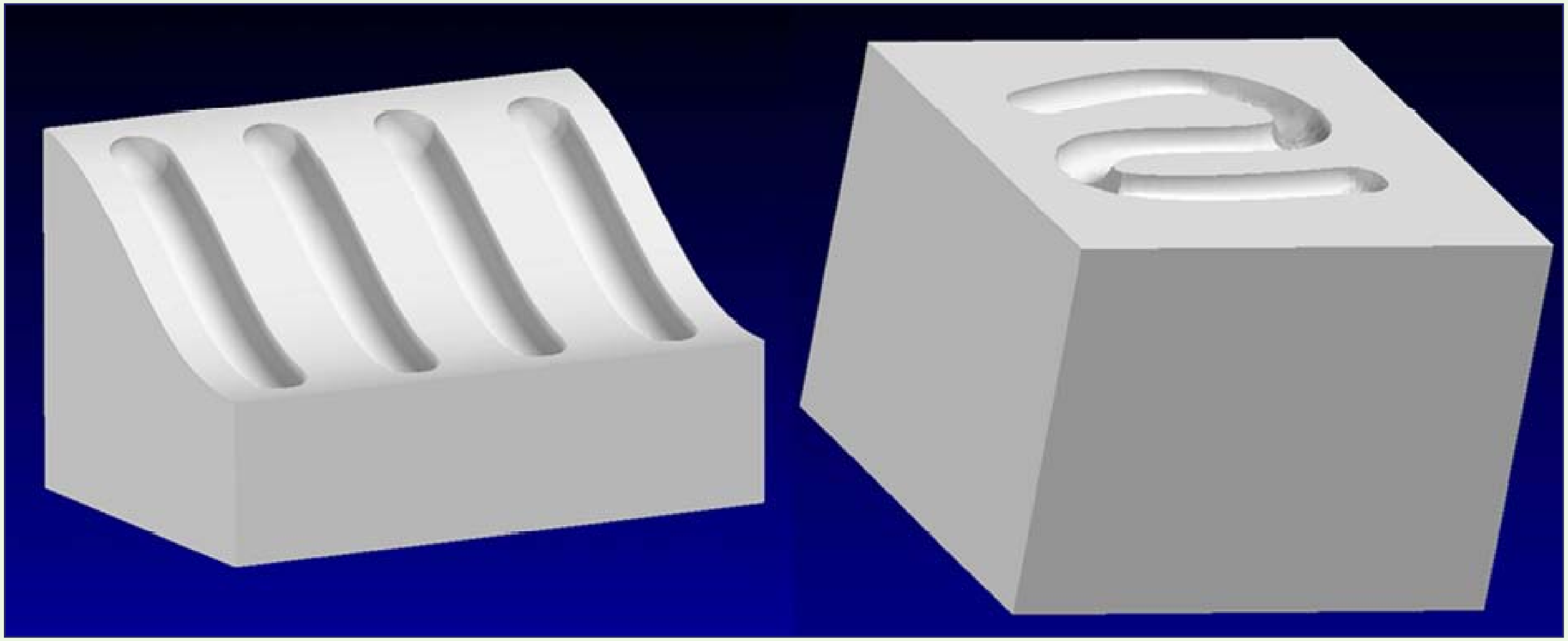
Secondary 3D model of representation

- The last step of the erosion is the generation of the erosion surface from the arches of dilation.
- This surface can be obtained triangulating point-to-point the consecutives arches of the trajectory (they were generated in an orderer manner).
- The joining of consecutive arches is accurate and fast, since most graphic cards are optimized for drawing triangles.
- The erosion surfaces are finally added to the previously obtained cut surfaces and a portion of the original surfaces.



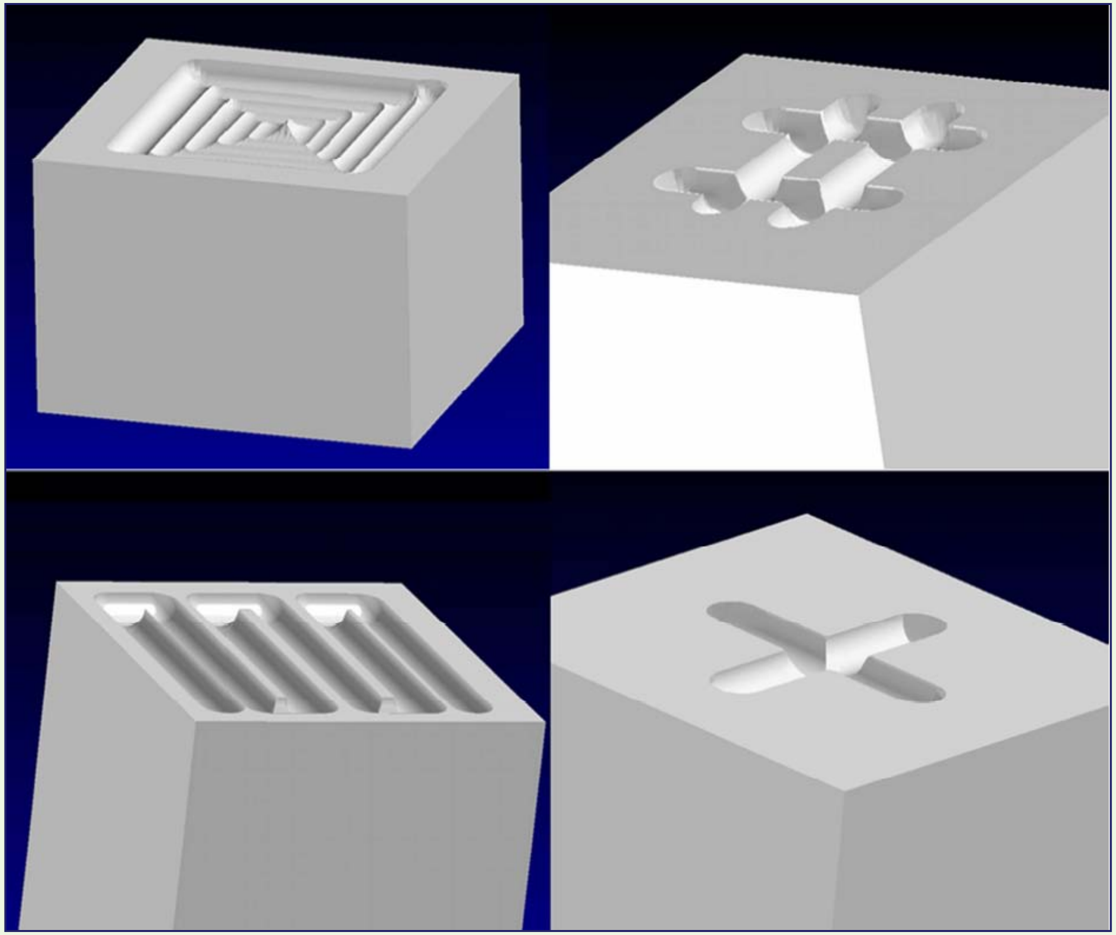
Secondary 3D model of representation

- 3D Regularized partial erosion examples:



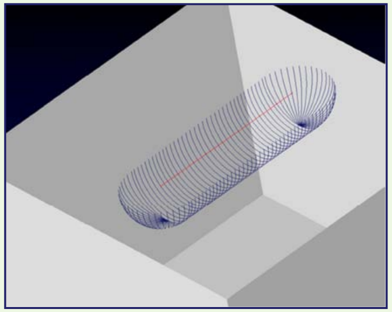
Secondary 3D model of representation

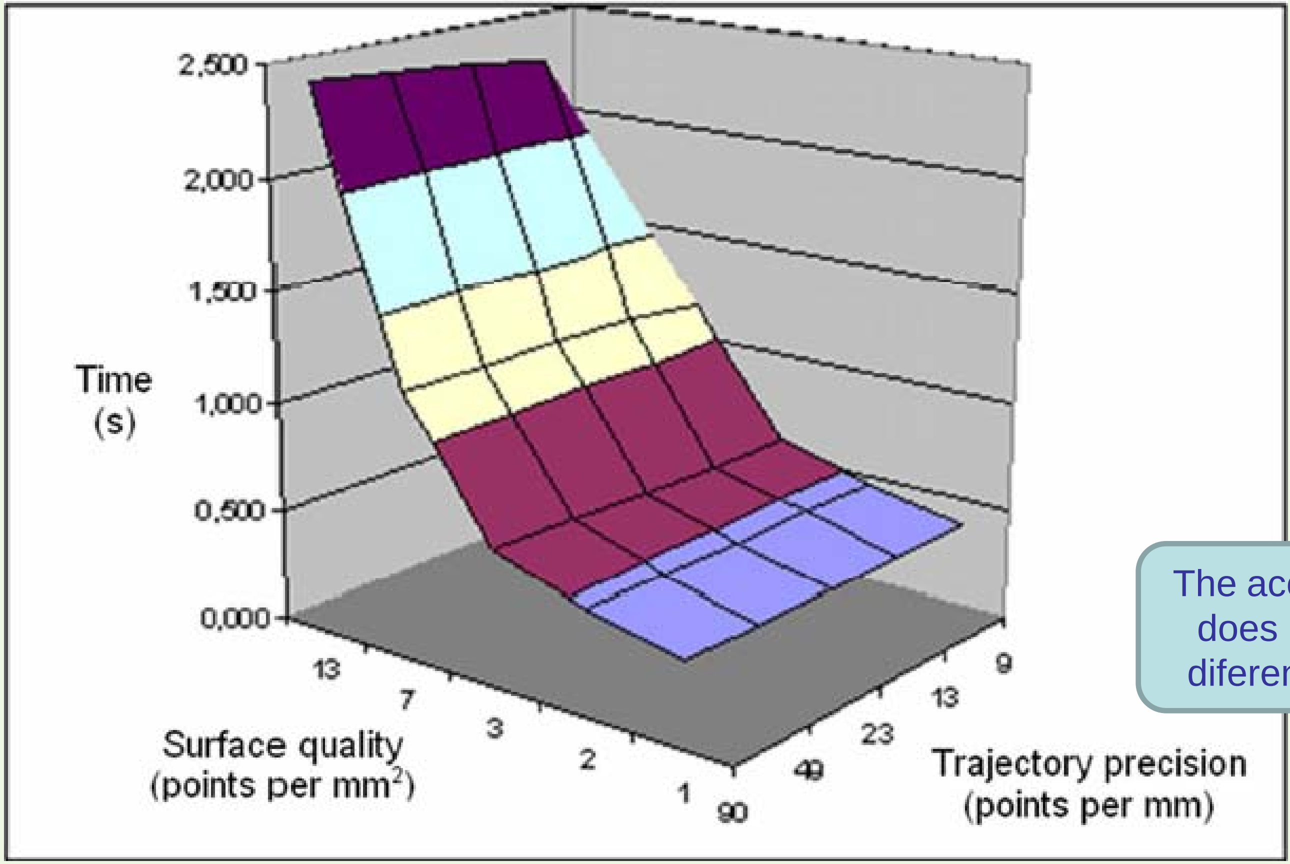
- 3D Regularized partial erosion examples:



Secondary 3D model of representation

- The process of displaying the machined object has a low temporal cost.
- Execution time for a simple object
- Pentium IV 1.8MHz, 2GB RAM, Windows XP.





The accuracy of the trajectory does not cause substantial diferences in execution time

Conclusions

- The proposed model gives solution to the design of complex objects and an arithmetic support to the generation of machining trajectories, which are already defined in the design phase.
- The model is based on Mathematical Morphology and Set Theory, and all the operation are done exclusively between point sets (no solids/surfaces).
- The model is generic, since the str.elem. is any point subset (in fact, it was extended to squared-end tools, corner radius tools, toroidal, and conical).
- The model is robust:
  - Every piece is treated as a set of point, so no complex calculations are required (instant basic operations just need a transform and the Euclidean distance).
  - The surface calculation required to display the objects is done by a simple triangulation method that homogeneously deals with every point and that avoids problematic cases or incorrect solutions.
- A main value of the model is that the calculation of the erosion surface gives a direct visualization of the object, so there is no intermediate steps between the erosion calculation and the display.

Conclusions

- The model succeeds in a faithful and very quick representation of the eroded object.
- The user can check the validity of the object modeling, since the model exactly represents the result of the machining process.
- The generation of the erosion surface by the triangulation of the trajectory points eases the display optimization and the use of textures.
- It offers relevant functions, such as wire representation and face occlusion.
- Since the user can control the definition of the surfaces, it is possible to identify specific areas which may require a greater precision.
- In a future research, it is planned to deal with more complex str.elem. and with the application of the proposed model on current CAD/CAM systems.

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